Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|

Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

**Time** 1 hour 30 minutes

**Paper reference** **WFM02/01**

# Mathematics

## International Advanced Subsidiary/Advanced Level
## Further Pure Mathematics F2

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
  Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over* ▶

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P 7 1 1 0 1 A 0 1 3 2

Pearson

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**1.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Express the complex number

$$-4 - 4\sqrt{3}\mathrm{i}$$

in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$

**(3)**

(b) Solve the equation

$$z^3 + 4 + 4\sqrt{3}\mathrm{i} = 0$$

giving your answers in the form $re^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$

**(4)**

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**Question 1 continued**

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**Question 1 continued**

Leave blank

**Question 1 continued**

**(Total 7 marks)**

**Q1**

Leave blank

**2.** Determine the general solution of the differential equation

$$2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 5\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 2\mathrm{e}^{3x}$$

**(6)**

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**Question 2 continued**

**(Total 6 marks)**

**Q2**

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**3.**

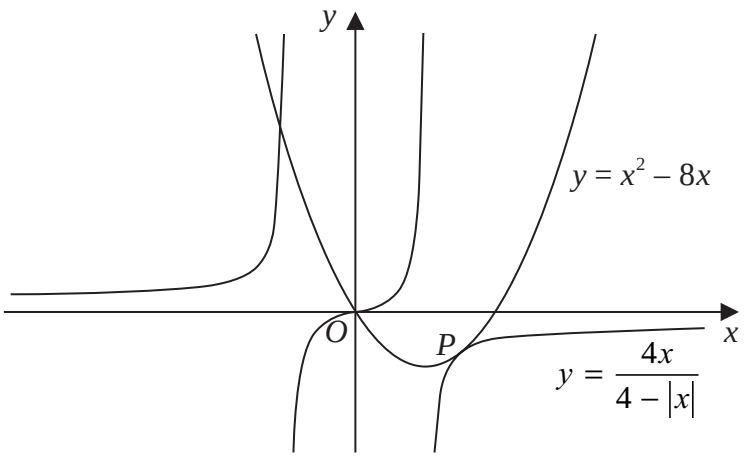


**Figure 1**

Figure 1 shows a sketch of the curve $C_1$ with equation

$$y = \frac{4x}{4 - |x|}$$

and the curve $C_2$ with equation

$$y = x^2 - 8x$$

For $x > 0$, $C_1$ has equation $y = \frac{4x}{4 - x}$

(a) Use algebra to show that $C_1$ touches $C_2$ at a point $P$, stating the coordinates of $P$

**(5)**

(b) Hence or otherwise, using algebra, solve the inequality

$$x^2 - 8x > \frac{4x}{4 - |x|}$$

**(6)**

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**Question 3 continued**

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**Question 3 continued**

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**Question 3 continued**

**(Total 11 marks)**

**Q3**

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**4.**

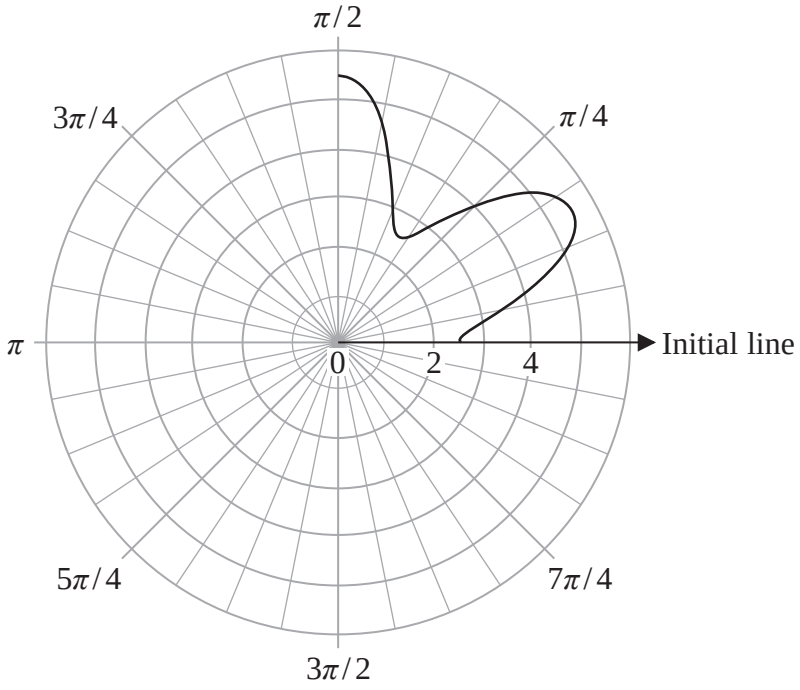


**Figure 2**

Figure 2 shows part of the curve with polar equation

$$r = 4 - \frac{3}{2}\cos 6\theta \qquad 0 \leqslant \theta < 2\pi$$

(a) Sketch, on the polar grid in Figure 2,

(i) the rest of the curve with equation $r = 4 - \frac{3}{2}\cos 6\theta \qquad 0 \leqslant \theta < 2\pi$

(ii) the polar curve with equation $r = 1 \qquad 0 \leqslant \theta < 2\pi$

**A spare copy of the grid is given on page 15.**

**(3)**

**In part (b) you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(b) Determine the exact area enclosed between the two curves defined in part (a).

**(7)**

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**Question 4 continued**

Leave blank

**Question 4 continued**

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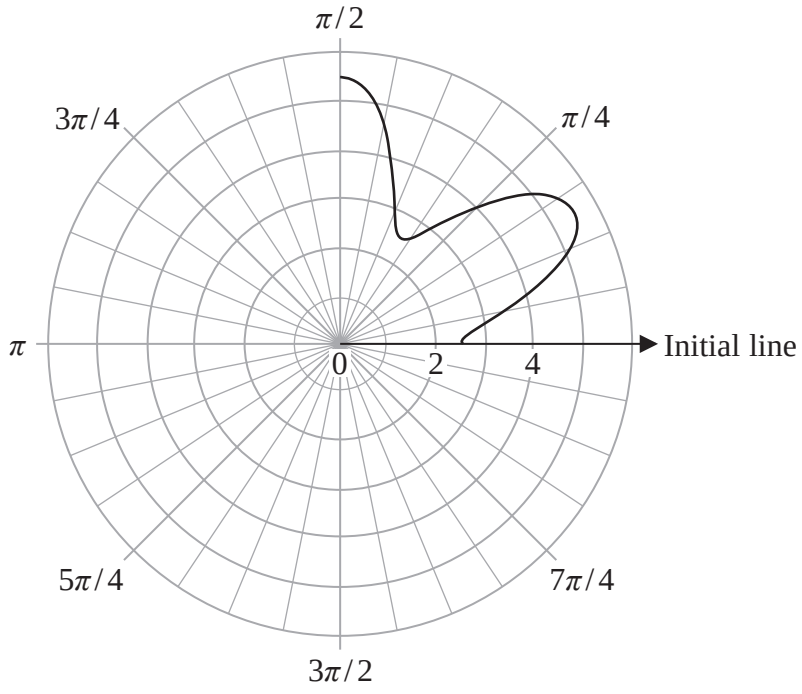
**Question 4 continued**

**Only use this grid if you need to redraw your answer to part (a)**




**Copy of Figure 2**

**(Total 10 marks)**

**Q4**

Leave blank

**5.** $y = \sqrt{4 + \ln x} \qquad x > \frac{1}{2}$

(a) Show that

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\frac{9 + 2\ln x}{4x^2(4 + \ln x)^{\frac{3}{2}}}$$

**(5)**

(b) Hence, or otherwise, determine the Taylor series expansion about $x = 1$ for $y$, in ascending powers of $(x - 1)$, up to and including the term in $(x - 1)^2$, giving each coefficient in simplest form.

**(3)**

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**Question 5 continued**

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**Question 5 continued**

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**Question 5 continued**

**Q5**

**(Total 8 marks)**

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**6.** Given that $A > B > 0$, by letting $x = \arctan A$ and $y = \arctan B$

(a) prove that

$$\arctan A - \arctan B = \arctan\left(\frac{A - B}{1 + AB}\right)$$

**(3)**

(b) Show that when $A = r + 2$ and $B = r$

$$\frac{A - B}{1 + AB} = \frac{2}{(1 + r)^2}$$

**(2)**

(c) Hence, using the method of differences, show that

$$\sum_{r=1}^{n} \arctan\left(\frac{2}{(1 + r)^2}\right) = \arctan(n + p) + \arctan(n + q) - \arctan 2 - \frac{\pi}{4}$$

where $p$ and $q$ are integers to be determined.

**(4)**

(d) Hence, making your reasoning clear, determine

$$\sum_{r=1}^{\infty} \arctan\left(\frac{2}{(1 + r)^2}\right)$$

giving the answer in the form $k\pi - \arctan 2$, where $k$ is a constant.

**(2)**

Leave blank

**Question 6 continued**

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**Question 6 continued**

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**Question 6 continued**

**(Total 11 marks)**

**Q6**

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**7.** A transformation from the $z$-plane to the $w$-plane is given by

$$w = \frac{(1 + \mathrm{i})z + 2(1 - \mathrm{i})}{z - \mathrm{i}} \qquad z \neq \mathrm{i}$$

The transformation maps points on the imaginary axis in the $z$-plane onto a line in the $w$-plane.

(a) Find an equation for this line.

**(2)**

The transformation maps points on the real axis in the $z$-plane onto a circle in the $w$-plane.

(b) Find the centre and radius of this circle.

**(6)**

Leave blank

**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

**(Total 8 marks)**

**Q7**

Leave blank

**8.** (a) Show that the transformation $v = y - 2x$ transforms the differential equation

$$\frac{dy}{dx} + 2yx(y - 4x) = 2 - 8x^3 \qquad \text{(I)}$$

into the differential equation

$$\frac{dv}{dx} = -2xv^2 \qquad \text{(II)}$$

**(4)**

(b) Solve the differential equation (II) to determine $v$ as a function of $x$

**(4)**

(c) Hence obtain the general solution of the differential equation (I).

**(1)**

(d) Sketch the solution curve that passes through the point $(-1, -1)$.

On your sketch show clearly the equation of any horizontal or vertical asymptotes.

You do **not** need to find the coordinates of any intercepts with the coordinate axes or the coordinates of any stationary points.

**(5)**

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**Question 8 continued**

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**Question 8 continued**

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**Question 8 continued**

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**Question 8 continued**

**Q8**

**(Total 14 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**